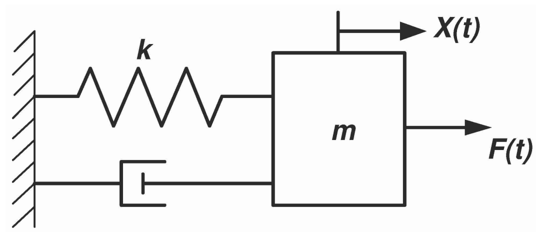
ME 120: Linear Systems & Control (Fall 2020)

Homework #7

Due 12/4/2020 at midnight Pacific Time via iLearn

* *Collaborating and working with your peers are encouraged for homeworks, but copying is not allowed and you have to turn in your own independent writing.*

Consider the same MSD system in Example 2.7.1



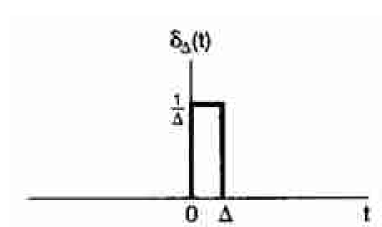
In this homework, we will see what this system does to input signals $u(t) = F(t)$. Throughout, let

$$m = 10^{-3}\ \text{kg}, \qquad k = 1\ \text{kg/s}, \qquad b = 0.1\ \text{kg/s}^2$$

1. Recall that the Fourier transform $F(j\omega)$ of a signal $f(t)$ is its Laplace transform $F(s)$ when the complex variable s is purely imaginary. Unlike s which was a complex-valued "frequency" and hence a bit unclear, ω is real-valued and a more intuitive notion of frequency. When applied to the system's impulse response $h(t)$, the resulting $H(j\omega)$ is called the system's "frequency response".
   - (i) Compute the frequency response of the above MSD system. (1 points)
   - (ii) Compute the modulus of the frequency response, $|H(j\omega)|$, and plot it for $\omega \in [0, 100]$. How does it differ between low and high frequencies? (3 points)
   - (iii) Compute the system's impulse response $h(t)$ over $t \in [0, 1]$ with a sampling frequency of 1 kHz (*not* 1 Hz). Plot the response. (1 point)
   - (iv) Load the input signal $u(t)$ from `hw7_data.mat`. Compute the system's response $y(t)$ to this input, and plot both u and y in the same figure. What has been the effect of the system on u? How does this relate to the system's frequency response from part (ii)? (3 points)
     Hint: use MATLAB's function `conv` for computing the system's response, but be careful that it only approximates the $\int_0^t h(t-\tau)u(\tau)$ part of the $h \star u = \int_0^t h(t-\tau)u(\tau)d\tau$ using a sum, so you have to manually include the $d\tau$ part.

2. Recall that the impulse function has a unit area under its curve but all of that area is accumulated at $t = 0$. Therefore, the impulse function is often approximated by

$$\delta_\Delta(t) = \begin{cases} 1/\Delta & \text{for } 0 \le t \le \Delta \\ 0 & \text{for otherwise} \end{cases}$$



   when $\Delta \to 0$.

Consider the same MSD system above. Using MATLAB's `ode45`, compute the response of the system of $u(t) = \delta_\Delta(t)$ for $\Delta = 0.5, 0.1, 0.01$. Plot all three of them together with the theoretical impulse function $h(t)$ in the same plot. Use `legend()` to distinguish the curves. Explain the results. (2 points)

Hint: When you are writing the function `odefun` for using with `ode45`, recall that it has to have the form `function dx = odefun(t, x)`. This function is precisely the function $f(t, x)$ in

$$\dot{\mathbf{x}} = f(t, \mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{B}u(t)$$

So when writing `odefun`, use its input `t` to check if `t >= 0 && t <= Delta` or not, and set the value of `u_t` accordingly.